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UDC 534

ON THE CORRECTNESS OF THE APPROXIMATE INVESTIGATION OF SYNCHRONOUS MACHINE ROTOR SWINGING

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We consider the complete system of equations for the dynamics of a synchronous machine with two windings on the rotor. We indicate the conditions under which the original system of equations can be reduced to the equation of motion of the rotor. The conditions for rotor selfoscillations to arise are determined as a result of investigating this equation. The complete system of equations for the dynamics of a synchronous machine containing equations describing the electrical responses and equations for the rotor's mechanical motion are obtained in [1]. Transient responses in electric circuits were investigated next, as was the expression for the electromechanical moment under a constant rotation velocity of a rotor with one circuit, e. g., field winding. However, in many of the later works the electrical equations were used only for finding the electromechanical moment under a constant spin rate of the rotor, and the problem was then reduced to the study of the equation for the rotor's mechanical motion [2, 3]. Here the conditions for which such an analysis is admissible were not mentioned. It was established that the swinging of a synchronous machine's rotor can be revealed in the form of selfoscillations. Vlasov [4] has investigated the equation of motion of a rotor and, under the assumption of a small parameter in the first derivative term, has found the conditions for the excitation of selfoscillations. Investigation in this same direction was carried out in [5]. However, in the investigation of the selfoscillations Vlasov did not examine the responses in the electrical circuits, while the expression for the electromechanical moment was obtained from power considerations. Other works have used particular expressions for the electromechanical moment, which can not explain the selfoscillation phenomenon.

1. Equations for synchronous machine dynamics. The equations for the dynamics of a salient-pole synchronous machine with two rotor windings — the field winding and the quadrature damping winding — under the assumption made in [1] have the form

$$L_0 q_0^{\cdot\cdot} + R q_0^{\cdot} = -u_0 \tag{1.1}$$

$$L_x q_x^{\cdot\cdot} + L_y q_y^{\cdot} \varphi^{\cdot} + M_x q_4^{\cdot\cdot} + M_y q_5^{\cdot} \varphi^{\cdot} + R q_x^{\cdot} = -E_x$$

$$-L_x q_x^{\cdot} \varphi^{\cdot} + L_y q_y^{\cdot\cdot} - M_x q_4^{\cdot} \varphi^{\cdot} + M_y q_5^{\cdot\cdot} + R q_y^{\cdot} = -E_y$$

$$L_4 q_4^{\cdot\cdot} + {}^3/_2 M_x q_x^{\cdot\cdot} + R_4 q_4^{\cdot} = E_4$$

$$L_5 q_5^{\cdot\cdot} + {}^3/_2 M_y q_y^{\cdot\cdot} + R_5 q_5^{\cdot} = 0$$

$$J\varphi^{\cdot\cdot} = {}^3/_2 \left[M_y q_x^{\cdot} q_5^{\cdot} - M_x q_y^{\cdot} q_4^{\cdot} - (L_x - L_y)\, q_x^{\cdot}\, q_y^{\cdot}\right] + T$$

Here φ is the angle between the rotor's longitudinal axis and the magnetic axis of the stator's first phase, $q_x^{\cdot}$, $q_y^{\cdot}$ are the stator's longitudinal and transverse currents, $q_4^{\cdot}, q_5^{\cdot}$ are the currents in the rotor windings, E_x, E_y are the longitudinal and transverse components of the circuit voltage applied to the stator, E_4 is the excitation e. m. f. (electromotive force), L_x, L_y are the selfinductance coefficients in the machine's longitudinal and transverse axes, L_4, L_5 are the self-inductance coefficients in the rotor windings, M_x, M_y are the mutual-inductance coefficients of the stator's phases with the rotor windings, R, R_4, R_5 is the active resistance of the stator's phase and the rotor windings, J is the rotor's moment of inertia, T is the moment of the external mechanical forces applied to the rotor's shaft. The first equation in system (1.1) describes the change in the stator's null current. The variable $q_0^{\cdot}$ does not enter into the remaining equations of system (1.1); therefore, we can exclude this equation from further consideration.

We introduce the dimensionless parameters and the dimensionless variables

$$\alpha_1 = \frac{L_4}{L_x}, \quad \beta_1 = \frac{R_4}{\omega_1 L_x}, \quad \gamma_1 = \frac{M_x}{L_x}, \quad \gamma_1^{\circ} = \frac{3}{2}\gamma_1$$

$$\alpha_2 = \frac{L_5}{L_x}, \quad \beta_2 = \frac{R_5}{\omega_1 L_x}, \quad \gamma_2 = \frac{M_y}{L_x}, \quad \gamma_2^{\circ} = \frac{3}{2}\gamma_2$$

$$\lambda = \frac{L_y}{L_x}, \quad \xi = \frac{R}{\omega_1 L_x}, \quad \eta = \frac{E_4}{E_0}, \quad M_0 = \frac{3}{2}\frac{E_0^2}{J L_x \omega_1^4}, \quad T_0 = \frac{T}{J\omega_1^2}$$

$$\tau = \omega_1 t, \quad \theta = \frac{\omega_2}{\omega_1}\tau - \tau - \frac{\pi}{2}, \quad s = \theta^{\cdot} = \frac{\omega_2}{\omega_1} - 1, \quad \frac{\omega_2}{\omega_1} = \frac{d\varphi}{d\tau}$$

$$q_x^{\cdot} = \frac{E_0}{L_x\omega_1^2}x, \quad q_y^{\cdot} = \frac{E_0}{L_x\omega_1^2}y, \quad q_4^{\cdot} = \frac{E_0}{L_x\omega_1^2}u, \quad q_5^{\cdot} = \frac{E_0}{L_x\omega_1^2}v$$

where ω_1 is the frequency of the external circuit, θ is the angle between the rotor's transverse axis and the rotating vector of the external circuit's e. m. f. Then the system of equations for the synchronous machine's transient modes reduces to the dimensionless form

$$x^{\cdot} + \xi x + \lambda(1+s)\,y + \gamma_1 u^{\cdot} + \gamma_2(1+s)\,v = \sin\theta \tag{1.2}$$

$$-(1+s)\,x + \lambda y^{\cdot} + \xi y - \gamma_1(1+s)\,u + \gamma_2 v^{\cdot} = -\cos\theta$$

$$\gamma_1^{0} x^{\cdot} + \alpha_1 u^{\cdot} + \beta_1 u = \eta, \quad \gamma_2^{\circ} y^{\cdot} + \alpha_2 v^{\cdot} + \beta_2 v = 0$$

$$\theta^{\cdot} = s, \qquad s^{\cdot} = T_0 + M_0 M$$

$$(M = [\gamma_2 x v - \gamma_1 y u - (1-\lambda)\,xy])$$

Here and subsequently the dot denotes differentiation with respect to the dimensionless time τ, and M is the moment of the electromagnetic forces referred to M_0.

2. Investigation of the expression for electromechanical moment for a constant spin rate of the rotor. For a constant spin rate of the rotor the first four equations of system (1.2) form an inhomogeneous system of linear differential equations with constant coefficients. The characteristic equation of this system satisfies the Routh-Hurwitz conditions when $\lambda\alpha_2 - \gamma_2\gamma_2^\circ > 0$, $\alpha_1 - \gamma_1\gamma_1^\circ > 0$, which always hold since these expressions are the system's transverse and longitudinal leakage coefficients [1]; therefore, the system's natural solutions decay. By virtue of this we shall neglect the transient values of the variables when investigating the electromechanical moment. The forced solutions of the system (the steady-state values of the variables) have the form

$$x = z_1,\ y = -z_2,\ u = z_3,\ v = -z_4 \tag{2.1}$$

$$z_j(\theta) = \frac{1}{a^2+b^2}\{[a(a_{1j}-b_{2j}) + b(b_{1j}+a_{2j})]\sin\theta +$$
$$[a(b_{1j}+a_{2j}) - b(a_{1j}-b_{2j})]\cos\theta\} + \eta\frac{A_{3j}(0)}{\Delta(0)} \quad (j = 1, 2, 3, 4)$$
$$a = \operatorname{Re}\Delta(p), \quad b = \operatorname{Im}\Delta(p), \quad a_{kj} = \operatorname{Re}A_{kj}(p), \quad b_{kj} = \operatorname{Im}A_{kj}(p)$$
$$(p = is, \quad i = \sqrt{-1})$$

Here $\Delta(p)$ is the determinant of system (1.2), $A_{kj}(p)$ are the minors of the elements of determinant $\Delta(p)$. Substituting (2.1) into the expression for M (in parantheses in (1.2)), we obtain the synchronous machine's moment with $s = \text{const}$

$$M = a_1(s)\eta^2 + b_1(s) + b_2(s)\cos 2\theta + b_3(s)\sin 2\theta + \eta c_1(s)\cos\theta + \eta c_2(s)\sin\theta \tag{2.2}$$

The quantity M has a constant component and a periodically-varying component. Under a synchronous rotation of the rotor $(s = 0)$ the first component called the braking torque, has the form

$$M_1 = a_1(0)\eta^2 + b_1(0) \tag{2.3}$$
$$a_1(0) = -\frac{\xi\gamma_1^2(\lambda^2+\xi^2)}{\beta_1^2(\lambda+\xi^2)^2}, \qquad b_1(0) = -\frac{1}{2}\frac{\xi(1-\lambda)^2}{(\lambda+\xi^2)^2}$$

The quantity M_1 depends only on the machine's parameters and is independent of the rotor's position. The braking torque obtains under any mode of operation and opposes the rotor's rotation. It vanishes when there are no ohmic losses in the stator circuit $(\xi = 0)$.

The second component (the synchronous torque) has the form

$$M_2 = b_2(0)\cos 2\theta + b_3(0)\sin 2\theta + \eta c_1(0)\cos\theta + \eta c_2(0)\sin\theta \tag{2.4}$$
$$b_2(0) = \frac{\xi(1-\lambda^2)}{2(\lambda+\xi^2)^2}, \qquad b_3(0) = \frac{(1-\lambda)(\xi^2-\lambda)}{2(\lambda+\xi^2)^2}$$
$$c_1(0) = \frac{\xi\gamma_1}{\beta_1(\lambda+\xi^2)^2}(\xi^2 - \lambda + 2\lambda^2)$$
$$c_2(0) = \frac{\gamma_1}{\beta_1(\lambda+\xi^2)^2}(\xi^2\lambda - \lambda^2 - 2\xi^2)$$

The synchronous torque arises as a result of the interaction of the rotor's constant magnetic field and the stator's rotating magnetic field. When there is no excitation ($\eta = 0$) the synchronous torque is caused by the difference in the inductance along the longitudinal and transverse axes and has the form

$$M_2 = \frac{1}{2}\frac{1-\lambda}{(\lambda+\xi^2)^2}[(\xi^2-\lambda)\sin 2\theta + \xi(1+\lambda)\cos 2\theta]$$

When the rotor windings have identical inductances along the longitudinal and transverse axes ($\lambda = 1$), the synchronous torque has the form

$$M_2 = \frac{\eta\gamma_1}{\beta_1(1+\xi^2)}[\xi\cos\theta - \sin\theta]$$

When there are no ohmic losses in the stator circuit ($\xi = 0$)

$$M_2 = -\frac{1}{2}\frac{1-\lambda}{\lambda}\sin 2\theta - \eta\frac{\gamma_1}{\beta_1}\sin\theta \qquad (2.5)$$

and if the inductances along the axes are equal ($\lambda = 1$), then only the second term in this expression remains.

The braking torque (2.3) and the synchronous torque (2.4) considered above completely determine the machine's electromechanical moment under a synchronous mode of operation ($s = 0$). These torques agree, as was to be expected, with those obtained in [1] for a machine with one winding on the rotor, because under synchronous rotation the presence of a damping winding does not affect the electromechanical moment.

For $s \neq 0$, i.e. for an asynchronous rotor spinning, the electromechanical moment is of form (2.2). The quantity $a_1(s)\eta^2 + b_1(s)$ in expression (2.2) is the mean value of the asynchronous moment over the period of slippage. The asynchronous moment is caused by the interaction of the stator's magnetic field with the field of currents induced in the rotor windings.

Let us derive the conditions under which expression (2.2) turns into the well-known expressions for the electromechanical moment, which have been used by various authors [1 – 3, 6 – 8]. We note that the ohmic resistance in the stator circuit is usually neglected when deriving the formula for the electromechanical moment. Therefore, if we set $\xi = 0$ in (2.2), we obtain

$$a_1(s) = 0, \quad c_1(s) = 0, \quad c_2(s) = -\gamma_1/\beta_1 \qquad (2.6)$$

$$b_{1,2}(s) = -\frac{s}{2}\left[\frac{\beta_2\gamma_2\gamma_2^\circ}{\lambda^2\beta_2^2 + s^2(\gamma_2\gamma_2^\circ - \lambda\alpha_2)^2} \pm \frac{\beta_1\gamma_1\gamma_1^\circ}{\beta_1^2 + s^2(\gamma_1\gamma_1^\circ - \alpha_1)^2}\right]$$

$$b_3(s) = -\frac{1}{2}\left[\frac{\lambda\beta_2^2 - \alpha_2 s^2(\gamma_2\gamma_2^\circ - \lambda\alpha_2)}{\lambda^2\beta_2^2 + s^2(\gamma_2\gamma_2^\circ - \lambda\alpha_2)^2} - \frac{\beta_1^2 - \alpha_1 s^2(\gamma_1\gamma_1^\circ - \alpha_1)}{\beta_1^2 + s^2(\gamma_1\gamma_1^\circ - \alpha_1)^2}\right]$$

In the case of identical windings along the rotor's longitudinal and transverse axes, $\lambda = 1$, $\gamma_1 = \gamma_2 = \gamma$, $\beta_1 = \beta_2 = \beta$, $\alpha_1 = \alpha_2 = \alpha$ and the electromechanical moment has the form [8]

$$M = -\frac{\gamma\gamma^\circ\beta s}{\beta^2 + s^2(\gamma\gamma^\circ - \alpha)^2} - \eta\frac{\gamma}{\beta}\sin\theta$$

If in expressions (2.6) we neglect the terms containing s^2, considering s to be small, then the expression for the electromechanical moment agrees with that obtained in [6].

In the case of identical windings along the rotor axes we obtain the obvious expression [9]

$$M = -\eta \frac{\Upsilon}{\beta} \sin\theta - \frac{\Upsilon\Upsilon^\circ}{\beta} s$$

From (2.6) we can obtain expressions for the coefficients of the electromechanical moment in the case when there is one field winding on the rotor. In fact, taking the damping winding as open, i.e. setting $\gamma_2 = 0,\ \alpha_2 = 0$, we obtain

$$b_{1,2}(s) = \mp \frac{s}{2} \frac{\beta_1 \Upsilon_1 \Upsilon_1^\circ \eta}{\beta_1^2 + s^2 (\Upsilon_1 \Upsilon_1^\circ - \alpha_1)^2} \tag{2.7}$$

$$b_3(s) = -\frac{1}{2}\left[\frac{1}{\lambda} - \frac{\beta_1^2 - \alpha_1 s^2 (\Upsilon_1 \Upsilon_1^\circ - \alpha_1)}{\beta_1^2 + s^2 (\Upsilon_1 \Upsilon_1^\circ - \alpha_1)^2}\right]$$

In the case of a cylindrical rotor ($\lambda = 1$)

$$M = -\eta \frac{\Upsilon}{\beta} \sin\theta - \frac{s}{2} \frac{\beta \Upsilon\Upsilon^\circ}{\beta^2 + s^2 (\Upsilon\Upsilon^\circ - \alpha)^2} (1 - \cos 2\theta) - \frac{s^2}{2} \frac{\Upsilon\Upsilon^\circ (\Upsilon\Upsilon^\circ - \alpha)}{\beta^2 + s^2 (\Upsilon\Upsilon^\circ - \alpha)^2} \sin 2\theta \tag{2.8}$$

Expressions (2.7) and (2.8) agree with those obtained in [1]. Thus, Gorev's moments are obtained from (2.2) with $\xi = 0,\ \alpha_2 = 0,\ \gamma_2 = 0$, i.e. for a rotor with one winding and with the stator's ohmic resistance neglected. If in (2.8) we neglect terms containing s^2, then

$$M = -\eta \frac{\Upsilon}{\beta} \sin\theta - \frac{s}{2} \frac{\Upsilon\Upsilon^\circ}{\beta} (1 - \cos 2\theta) \tag{2.9}$$

3. Swinging of a synchronous machine's rotor. The dynamics of a synchronous machine is described by the sixth-order system (1.2) of nonlinear differential equations, which is considerably difficult to investigate. We indicate below the conditions under which the motion of a synchronous machine's rotor can be correctly described by a second-order equation with the use of the expression for the electromechanical moment, obtained for a constant spin rate of the rotor. These conditions reduce to the following:

1) the largest time constant of the electrical loops is much smaller than the time constant of mechanical motion (in general, this is satisfied for small electrical machines);

2) in the time interval T', somewhat exceeding the transient time in the electrical circuits, the spin rate of the rotor s changes little, i.e. $T's^\cdot \ll 1$.

The second condition signifies that in the time interval T' the resulting moment of the electromagnetic and mechanical forces is not able to make an essential change in the angular velocity of the rotor. Then in the time interval T' we can set $s = \text{const}$ in the first four equations of system (1.2) and, neglecting the natural solutions (the transient values of the variables), find the steady-state values of the currents. Let T'' be the time during which the mechanical transient response takes place, which is much greater than T' by virtue of the conditions we have adopted. Therefore, the time interval T'' be divided up into small intervals T' during each of which the rotor spin rate is constant. For $s = \text{const}$ the electromechanical moment depends only upon the obtained stready-state values of the currents. Then in any interval $T \geqslant T'' \gg T'$ the variation of the variables θ and s can be described by the last two equations of the system (1.2) in which the electromechanical moment is determined by expression (2.2)

$$\theta^{\cdot} = s,$$
$$s^{\cdot} = T_0 + a_1(s)\eta^2 + b_1(s) + b_2(s)\cos 2\theta + b_3(s)\sin 2\theta + \eta c_1(s)\cos\theta + \eta c_2(s)\sin\theta \quad (3.1)$$

Since when passing from the sixth-order system (1.2) to the second-order system (3.1) we assume that the electrical transients go "rapidly" while the mechanical ones (variation of θ and s) go "slowly", the reduction of the study of the synchronous machine dynamics to the study of a second-order differential equation proves to be correct if $\theta^{\cdot}$ is of the same order as $s^{\cdot}$. But since s varies slowly, i.e. $s^{\cdot}$ is assumed to be a small quantity, the quantity $s = \theta^{\cdot}$ also should be small of the order of $s^{\cdot}$.

A stable limit cycle of system (3.1) which includes the equilibrium state, corresponds to the selfoscillatory nature of the rotor's swinging. On the phase cylinder $(-\pi < \theta \leqslant \pi)$ system (3.1) has four equilibrium states, coordinates of which are defined by the equations

$$s = 0$$
$$T_0 + a_1(0)\eta^2 + b_1(0) + b_2(0)\cos 2\theta + b_3(0)\sin 2\theta + \eta c_1(0)\cos\theta + \eta c_2(0)\sin\theta = 0$$

The nature of the equilibrium states depends upon the roots $\lambda = {}^1/_2 d \pm {}^1/_2 \sqrt{d^2 + 4c}$ of the corresponding characteristic equation, where

$$c = -2b_2(0)\sin 2\theta_i + 2b_3(0)\cos 2\theta_i - \eta c_1(0)\sin\theta_i + \eta c_2(0)\cos\theta_i$$
$$d = a_1'(0)\eta^2 + b_1'(0) + b_2'(0)\cos 2\theta_i + b_3'(0)\sin 2\theta_i + \eta c_1'(0)\cos\theta_i + \eta c_2'(0)\sin\theta_i$$

The equilibrium states $(\theta_2, 0)$, $(\theta_4, 0)$ are saddles and for them $c > 0$; the equilibrium states $(\theta_1, 0)$, $(\theta_3, 0)$ (with $c < 0$) are nodes if $d^2 + 4c \geqslant 0$, and if $d^2 + 4c < 0$, they are foci. The nodes (foci) are stable for $d < 0$ and unstable for $d > 0$. The focus becomes a composite one for $d = 0$. The linear approximation, i.e. the study of the characteristic equation, is not sufficient to ascertain its stability and the question can be solved by Liapunov indices which are expressed in terms of the coefficients of the nonlinear equations and obtained by discarding all terms of higher than third order in the right-hand sides of the original equations. If the Liapunov index for the composite equilibrium state is nonzero, then it characterizes the stability of this equilibrium state as well as the boundary of the stability region $d = 0$ [10].

An electronic computer was used to find the equilibrium states of system (3.1) and to investigate their stability. The change in stability of the equilibrium state $(\theta_3, 0)$ as a function of the parameters $\lambda = L_y / L_x$, ξ (characterizing the ohmic losses in the stator curcuit), and T_0 (the mechanical moment on the rotor shaft) was traced. (The parameters λ and ξ occur in the coefficients of system (3.1)). The computations were carried out for $\alpha_1 = 1$, $\alpha_2 = 0.5$, $\beta_1 = 5$, $\beta_2 = 1$, $\gamma_1 = 0.3$, $\gamma_2 = 0.1$, $\eta = 0.1$ (here the synchronous machine has an underexcited magnetic system). Sections of the surface $d = 0$ by the planes $\lambda = \text{const}$ in the parameter space (T_0, λ, ξ) are shown in Fig. 1 for $\lambda = 0.1$ (a), $\lambda = 0.3$ (b), $\lambda = 0.5$ (c).

For $\xi = 0$ (no ohmic losses in the stator circuit) the equilibrium state $(\theta_3, 0)$ is stable $(d < 0)$. As ξ grows the equilibriun state becomes unstable $(d > 0)$ and then stable $(d < 0)$ once again. In the first case the Liapunov index is positive, while in the second case, negative. Consequently, the stability region boundary $d = 0$ for the equilibrium state $(\theta_3, 0)$ consists of two parts. One part of it is "dangerous" (curve *1*, Fig. 1). Upon intersecting it as ξ grows, a single unstable limit cycle shrinks to a stable equilibrium state. The equilibrium state becomes unstable. The second part of boundary $d = 0$ is "safe" (curve *2*, Fig. 1). Upon intersecting it as ξ decreases, a single stable limit cycle emerges from the stable equilibrium state, corresponding to a "soft" mode of excitation of selfoscillations [9]. The equilibrium state becomes unstable.

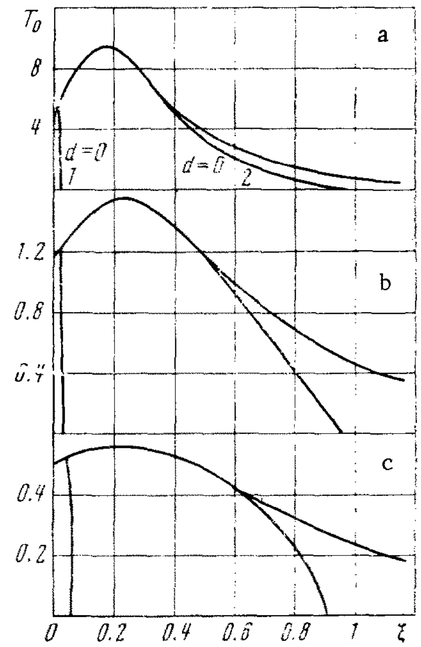

Fig. 1

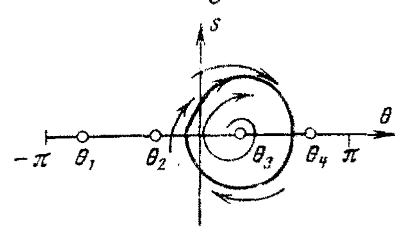

Fig. 2

An approximate integration of system (3.1) on an electronic computer [11] showed that for parameter values close to the "safe" part of boundary $d = 0$ there exists a stable limit cycle (Fig. 2) which grows as ξ decreases and then vanishes. (The nature of the cycle's disappearance was not investigated). The equilibrium state $(\theta_1, 0)$ also changes stability twice as ξ grows. With an increase in ξ or T_0 the foci $(\theta_1, 0)$ and $(\theta_3, 0)$ become nodes, then merge with the saddles $(\theta_2, 0)$ and $(\theta_4, 0)$, respectively, and vanish through composite equilibrium states of the "saddle-node" type (Fig. 1).

The investigation made permits the following conclusions: (a) under the conditions (1) and (2) indicated the swinging of a synchronous machine's rotor can be approximately described by the equation of mechanical motion of the rotor, in which is used the expression for the electromechanical moment, obtained for constant spin rate of the rotor; (b) a selfoscillation of the rotor is possible only in the presence of ohmic losses in the stator circuit.

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APPLICATION OF THE BUBNOV-GALERKIN PROCEDURE TO THE PROBLEM OF SEARCHING FOR SELFOSCILLATIONS

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We propose the use of the Bubnov-Galerkin procedure to the search for self-oscillations. We establish the existence and the convergence of the approximations. In the basic case we have obtained the asymptotics of the rate of convergence. In [1] it was shown, on the basis of the results in [2], how we can construct finite-dimensional approximations to the periodic solutions of autonomous systems. Below we have pointed out another approach to solving the approximation problem, based on the parameter functionalization method proposed in [3].

1. We first consider an autonomous system of ordinary differential equations

$$dx/dt = f(x) \qquad (x \in R^n) \tag{1.1}$$

where f is a continuously differentiable mapping of a region $G \subset R^n$ into R^n. We assume that in region G system (1.1) has an isolated cycle Γ whose smallest positive period is ω_0. Let $x_0 \in \Gamma$ and let $x^*(t)$ be the solution of system (1.1) with the initial condition x_0 at $t = 0$. We assume cycle Γ to be simple, i.e. unity is a simple eigenvalue of the translation operator at time ω_0 along the trajectories of the variational system